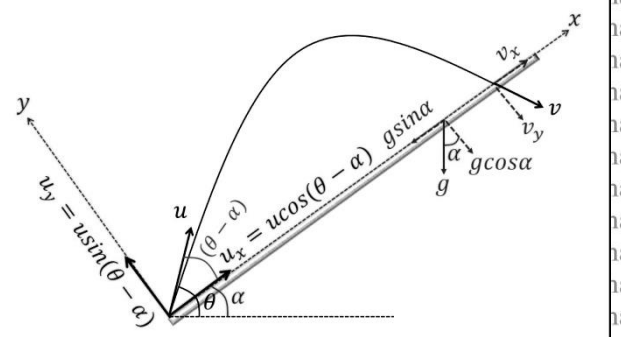
Projectile on inclined plane



(1)

On y- axis

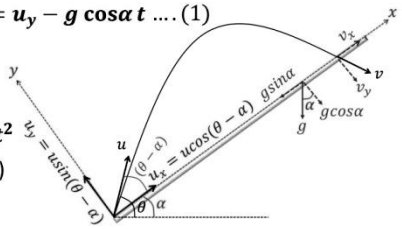
$[v = u + at] \Rightarrow \boldsymbol{v_y = u_y - g\,cos\alpha\, t}$ .... (1)

$\left[S = ut + \frac{1}{2}at^2\right]$

$\boldsymbol{y = u_y t - \frac{1}{2} g\,cos\alpha t^2}$ ..... (2)

$[v^2 = u^2 + 2as]$

$\boldsymbol{v_y^2 = u_y^2 - 2g cos\alpha . y}$ ..... (3)



(2)

To find of flight time (T):

From eq. (2)

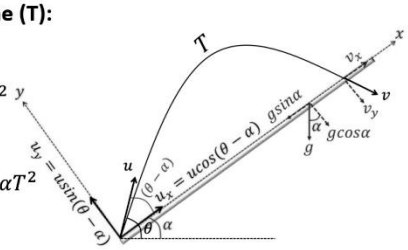
$y = u_y t - \frac{1}{2} g\,cos\alpha t^2$

$y = 0$ at $t = T$

$\Rightarrow 0 = u_y T - \frac{1}{2} g cos\alpha T^2$

$\Rightarrow u_y T = \frac{1}{2} g cos\alpha T^2$

$\Rightarrow T = \frac{2u_y}{g cos\alpha}$ $\quad \Rightarrow \boldsymbol{T = \frac{2usin(\theta-\alpha)}{gcos\alpha}}$



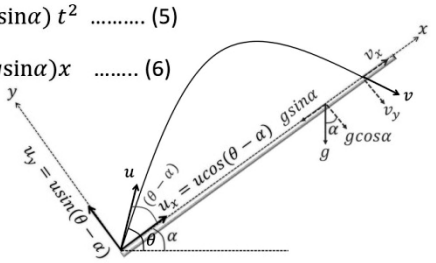
(3)

On x-axis:

$v_x = u_x - g\,sin\alpha\, t$ ..... (4)

$x = u_x t - \frac{1}{2}(gsin\alpha)\, t^2$ ..... (5)

$v_x^2 = u_x^2 - 2\,(gsin\alpha)x$ ..... (6)



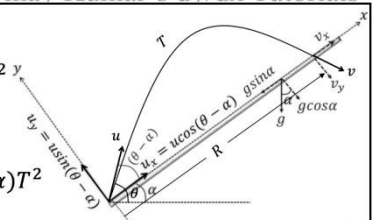
(4)

To find Range 'R'

$x = u_x t - \frac{1}{2}(g\,\sin\alpha)\, t^2$

$\boldsymbol{at\ t = T,} \quad \boldsymbol{x = R}$

$\Rightarrow R = u_x T - \frac{1}{2}(g\,\sin\alpha) T^2$



$\Rightarrow R = u\cos(\theta-\alpha)\left[\frac{2usin(\theta-\alpha)}{gcos\alpha}\right] - \frac{1}{2} g sin\alpha \left[\frac{2usin(\theta-\alpha)}{gcos\alpha}\right]^2$

$\Rightarrow R = \frac{2u^2 sin(\theta-\alpha)}{gcos\alpha}\left[\cos(\theta-\alpha) - \frac{sin\alpha\, sin(\theta-\alpha)}{cos\alpha}\right]$

(5)

$R = \frac{2u^2 sin(\theta-\alpha)}{gcos\alpha}\left[\cos(\theta-\alpha) - \frac{sin\alpha\, sin(\theta-\alpha)}{cos\alpha}\right]$

$= \frac{2u^2 sin(\theta-\alpha)}{gcos^2\alpha}[cos\alpha cos(\theta-\alpha) - sin\alpha sin(\theta-\alpha)]$

$= \frac{2u^2 sin(\theta-\alpha)}{gcos^2\alpha}\cos[\alpha + (\theta-\alpha)]$

$\Rightarrow \boldsymbol{R = \frac{2u^2 sin(\theta-\alpha)\cos\theta}{gcos^2\alpha}}$

(6)

$\boldsymbol{R_{max}}$ $\quad R = \frac{2u^2 sin(\theta-\alpha)\cos\theta}{gcos^2\alpha}$

$= \frac{u^2}{gcos^2\alpha}[sin\{(\theta-\alpha)+\theta\} + sin\{(\theta-\alpha)-\theta\}]$

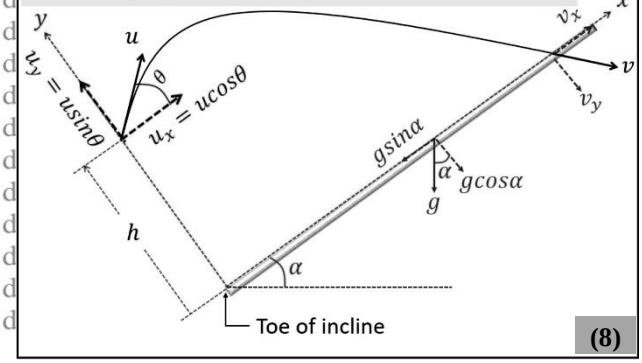
$\Rightarrow R = \frac{u^2}{gcos^2\alpha}[sin(2\theta-\alpha) - sin\alpha]$

For R_{max} $\quad sin(2\theta-\alpha) \Rightarrow$ *maximum*

$2\theta - \alpha = 90^o \Rightarrow 2\theta = 90^o + \alpha$ $\quad \Rightarrow \theta = 45^o + \frac{\alpha}{2}$

(7)

Projectile on incline whose point of projection is other than the toe of incline.



(8)

On y-axis⇨

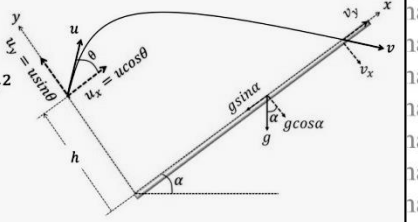
$v_y = u_y - g\cos\alpha.t$

$y = u_y t - \frac{1}{2} g\cos\alpha.t^2$

$v_y^2 = u_y^2 - 2g\cos\alpha.y$

To find flight time⇨

$-h = u_y T - \frac{1}{2} g\cos\alpha\, T^2 \quad \Rightarrow T = ?$



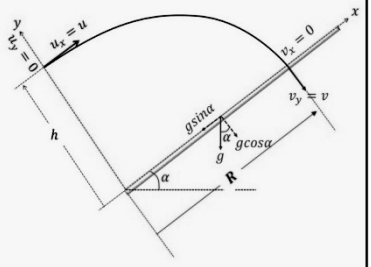
(9)

For range R⇨

$v_x^2 = u_x^2 - 2g\sin\alpha.x$

$\Rightarrow 0 = u^2 - 2g\sin\alpha.R$

$\Rightarrow R = \frac{u^2}{2g\sin\alpha}$



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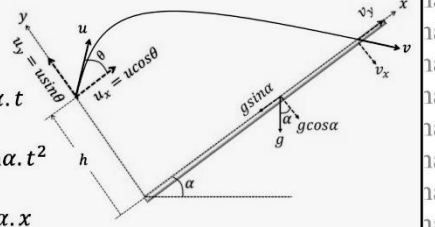
On x-axis⇨

$v_x = u_x - g\sin\alpha.t$

$x = u_x t - \frac{1}{2} g\sin\alpha.t^2$

$v_x^2 = u_x^2 - 2g\sin\alpha.x$



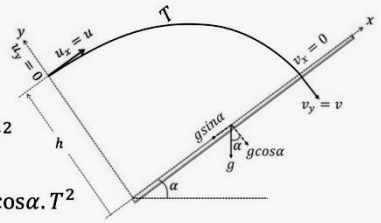
(10)

For h ⇨

$y = u_y t - \frac{1}{2} g\cos\alpha.t^2$

$\Rightarrow -h = 0 \times T - \frac{1}{2} g\cos\alpha.T^2$

$\Rightarrow -h = -\frac{1}{2} g\cos\alpha \left(\frac{u}{g\sin\alpha}\right)^2$ $\Rightarrow h = \frac{u^2\cos\alpha}{2g\sin^2\alpha}$



(14)

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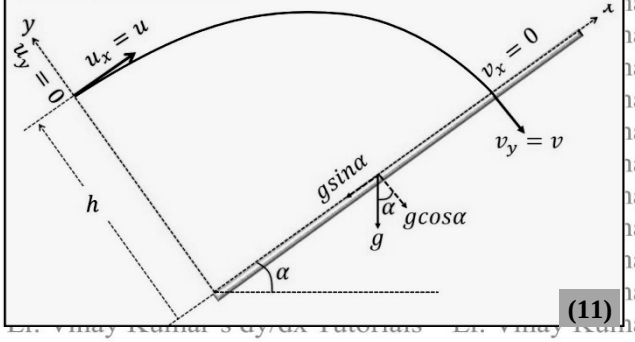
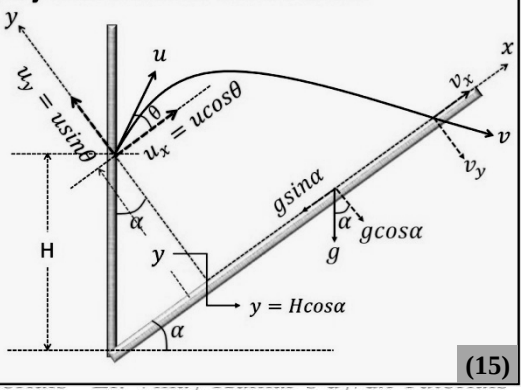
case-1: In a specific case in which the body is projected perpendicular to the first surface and it strikes the other surface perpendicular.



(11)

case-2: projectile between two surfaces:



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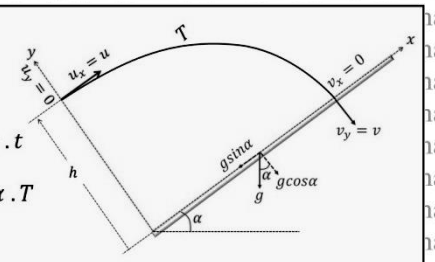
For flight time T⇨

$v_x = u_x - g\sin\alpha.t$

$\Rightarrow 0 = u - g\sin\alpha.T$

$\Rightarrow T = \frac{u}{g\sin\alpha}$



(12)

On y-axis

$v_y = u_y - g\cos\alpha.t$

$y = u_y t - \frac{1}{2} g\cos\alpha.t^2$

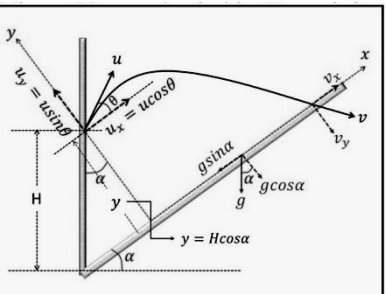
$v_y^2 = u_y^2 - 2g\cos\alpha.y$

On x-axis⇨

$v_x = u_x - g\sin\alpha.t$

$x = u_x t - \frac{1}{2} g\sin\alpha.t^2$

$v_x^2 = u_x^2 - 2g\sin\alpha.x$



(16)

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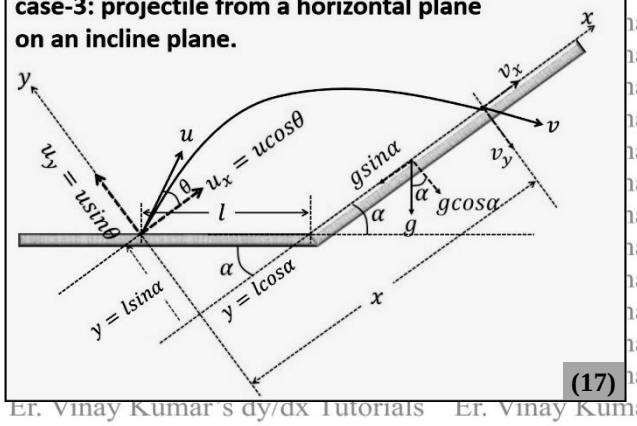
case-3: projectile from a horizontal plane on an incline plane.



(17)

On y-axis

$$v_y = u_y - g\cos\alpha.t$$

$$y = u_y t - \frac{1}{2}g\cos\alpha.t^2$$

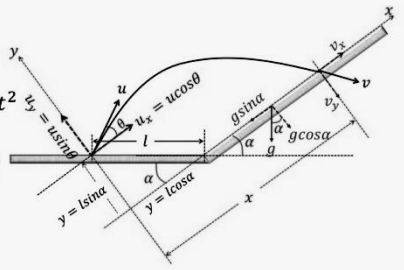
$$v_y^2 = u_y^2 - 2g\cos\alpha.y$$

On x-axis⇨

$$v_x = u_x - g\sin\alpha.t$$

$$x = u_x t - \frac{1}{2}g\sin\alpha.t^2$$

$$v_x^2 = u_x^2 - 2g\sin\alpha.x$$



(18)

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